Faith in the Future and Social Conflict: Economic Growth as a Mechanism for Political Stabilization

Alexander W. Bloedel

Professor Curtis Taylor, Faculty Advisor

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Abstract

This paper studies the mechanisms that link sociopolitical conflict and (expectations about) economic prosperity. Motivated by a large body of empirical and historical work on the correlation between economic development and democratization, I develop a game-theoretic model of economic growth with political economy constraints. In an economy where low-income agents are credit constrained, rapid and robust economic growth leads to increasing inequality early on, but provides the means to mitigate civil conflict when inequality becomes sufficiently large. The rate and persistence of growth similarly determines the stability of extant political institutions and the ability to transition from dictatorship to democracy.

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As we have seen from the experience of America and other industrialized countries... to the extent that the conflicts that emerge from modernization are over questions of "who gets what?" – in other words, to the extent that they are inherently subject to compromise, so that opposing interests can each come away with something positive in hand – the material gain that economic growth brings can help to resolve them. In doing so, growth not only relieves such tensions but also helps bind a society's competing groups together by fostering the sense that cooperation and compromise gain results.
– Benjamin Friedman, *The Moral Consequences of Economic Growth*

1 Introduction

The connection between economic development and political openness seems obvious upon casual observation: at present, the OECD countries are almost uniformly wealthy and democratic, while many underdeveloped nations in Latin America, sub-Saharan Africa, and the Middle East have political systems that are either plagued by perpetual conflict or, if stable, are decidedly non-democratic. Of course, countries that have *higher levels* of per capita wealth today must have *grown faster* in the past: virtually all of the global divergence in national wealth accrued over the last 200 years, since the Industrial Revolution began in Britain (Acemoglu (2009)). This suggests a link between political institutions and economic growth, which begs the questions: Do "good" political institutions cause growth? Does growth lead to the formation and endurance of "good" institutions? If so, what are the driving mechanisms?

The positive correlation – across time and across countries – between per capita wealth and political openness is one of the most optimistic empirical regularities in economics. It is also one of the most poorly understood, and – as the above questions suggest – has been the subject of heated debate in the literatures on growth, development, and political economy. The goal of this paper is to take a first step toward the theoretical analysis of this regularity from a somewhat novel perspective: that a nation's rate of economic growth has a positive first-order effect on its political stability and social cohesion. The theory that I develop is distinct from the well-known *modernization hypothesis* due to Lipset (1959), for "level effects" play essentially no role in my model. It also provides a more nuanced set of predictions about the conditions under which different political regimes – namely, authoritarian and democratic institutions – can remain effective and stable than the extant political economy theories of Acemoglu and Robinson (2006), on which this paper builds.

The conceptual framework that I use is largely indebted to the historical work of Friedman (2006). Through its positive effect on people's own sense of wellbeing – in particular, their *faith in the future* for themselves and their children – Friedman argues, "broad-based" economic growth is a definitive force for determining a nation's "moral" advancement: its degree of political openness and social cohesion, and, of particular importance, its ability to constructively overcome distributional conflict.[1] He attributes this to psychological factors, namely, that people judge their current economic status relative to two reference points: (1) their own past economic status and (2) the economic status of those around them. When growth is strong and people from large swaths of society feel as if they are "getting ahead" in the first sense, the second reference point becomes less salient. With its decreased importance, he argues, it is easier to overcome issues of distributional conflict and perhaps avoid them entirely.

While it is not difficult to imagine how such a theory could be formalized,[2] in this paper I remain in the standard domain of economic rationality. While incorporating such psychological elements is likely a fruitful area for future research,[3] the requisite modeling complications would distract from the main point. More importantly, I show that non-standard behavioral assumptions are not necessary to generate results along the lines imagined by Friedman.

In the model that I develop, an economy is populated by two different groups of agents who are divided along socioeconomic lines. Each group contributes to aggregate production, but may not be able to accumulate wealth at the same rate. Consistent with existing theory and empirical evidence on the evolution income distribution during the growth process[4] the difference in accumulation rates drives up inequality at early stages of development. As the aggregate economy grows and agents individually accumulate wealth, the critical issue is how gains from production are divided. Whichever group holds political

[1]Related mechanisms, particularly the impact of business cycle conditions on voting attitudes, have been discussed in the political science literature, e.g., Bloom and Price (1975).

[2]The incomplete contracting framework of Hart and Moore (2008) is a reasonable starting point.

[3]A handful of existing papers lie at the intersection of behavioral economics, political economy, and growth theory. Ortoleva and Bogliacino (2013) construct a growth model in which agents have preferences over their own consumption relative to the societal average and find that this reference dependence results in weakly higher growth rates. Cole et al (1992) study a related model in which wealth matters as a sign of social status, and hence agents have indirect utility functions over own wealth relative to a societal average. Unlike Ortovela and Bogliacino (2013), that paper links these indirect preferences to the specifics of an underlying social hierarchy. Benabou and Tirole (2006) and Benabou (2008) study models in which agents' optimally-distorted beliefs result in different preferences over redistribution and government size and can lead to multiple equilibria.

[4]See in particular Benabou (1996) for a comprehensive of the survey up to the date of publication.

power – the rich "elite" or the poor "masses" – determines the rule that governs this surplus allocation. When the elites control the political process, they rationally choose not to redistribute to the poor. Thus, inequality eventually reaches a critical level, at which point redistribution is necessary to maintain political stability. But, if the economy is not growing at a rapid enough pace at that time, sufficient redistribution may be impossible to prevent a "revolution." If the politically powerful elites cannot credibly commit to future redistribution, it is in their best interest to offer political control to the poor group – a process analogous to democratization. But, even under democracy, sufficient inequality and bad economic fundamentals can result in civil conflict analogous to the case of political revolution. I also discuss how the rate of growth affects the timing of of these events along the equilibrium path.[5]

The paper proceeds as follows. In the remainder of Section 1, I review the relevant empirical and theoretical literature. In Section 2, I solve a simple toy model that illustrates the main insights of the paper in the simplest possible form. Sections 3, 4, and 5 develop and solve the full model and present the main results. Finally, Section 6 concludes and discusses avenues for future research. All proofs are contained in the Appendix.

1.1 Literature Review

This paper is related to several distinct strands of literature, each with sizable empirical and theoretical components. In this subsection, I outline the papers most pertinent for the present analysis.

Tests of and Explanations for Modernization Theory: The first category consists of empirical evaluations of the modernization hypothesis and its variants. The primary question in this literature is whether per capita income levels have a causal effect on the level of democracy. Barro (1999) studies cross-country and within-country variation in democracy in the post-war data, finding that differences in per capita income can explain both. That is, not only are rich countries more democratic, but democratization in these

[5] I emphasize here – and in critical spots throughout the paper – that one must tread carefully when interpreting the economic environment and results of the present model. In particular, I routinely use (perhaps extreme) terms such as "revolution," "elites," "rich and poor agents," "dictatorship," and "democracy." In large part, this language is derived from the body of work surveyed in Acemoglu and Robinson (2006), and should therefore only be thought of as a homage to their terminology. More importantly, a fixed terminology is necessary for clarity and concision. I urge the reader to not read too heavily into these terms, as the economic environment that I develop is intentionally reduced-form. Throughout, I make note whenever multiple interpretations are available, and suggest which might be the most appropriate.

countries seems to have followed increases in wealth – precisely in line with modernization theory.

In two related papers, Acemoglu et al (2008, 2009) cast doubt on the modernization hypothesis. These papers are motivated by concerns about reverse causality and omitted variables. Acemoglu et al (2008) focus on the effect of income on the *level* of democracy, showing that the correlation between per capita income and level of democracy disappears when fixed effects or instrumental variables (IVs) are introduced into standard democracy regressions. The authors interpret these findings as evidence for their *critical junctures hypothesis*, which states that countries' political and economic development paths evolve jointly and that the contemporary distribution of wealth and democracy is the result of these joint dynamics reaching bifurcation points in the distant past. Another possible interpretation is that the causal effect of income on democracy operates on much longer time scales than previously thought (on the order of several centuries), since the income-democracy correlation re-appears when the sample reaches back to 1500 A.D. Using similar fixed-effects treatments, Acemoglu et al (2009) study the impact of income levels on *transitions* to and from democracy. In the post-war period, countries with per capita income above the world average in a given year are more likely to undergo democratization in the subsequent five years; countries with per capita incomes below the world average are more likely to experience transitions away from democracy. However, when the authors instead examine variations from the country's own average per capita income, the correlation with transitions toward and away from democracy almost entirely disappears. This is taken as further evidence against the modernization hypothesis.

Motivated by these studies, Benhabib et al (2011) find that the income-democracy correlation can be recovered even in the presence of fixed effects when the boundedness of the democracy indices is accounted for. As shown in Murtens and Wacziarg (2013), as early as the beginning of the twentieth century, the worldwide joint distribution of per capita income and democracy was largely bi-modal, with wealthy countries holding near perfect democracy scores. Because the democracy scores commonly used in the literature are bounded[6], it is natural to expect a flat correlation between income and democracy for already-wealthy countries – especially in the last 50-100 years. Murtens and Wacziarg (2013) show that, once initial level of democracy is included as a lagged variable in their

[6] There are several different democracy measures used in the empirical economics literature. These metrics are somewhat variable. Some measure democracy as a binary variable, while others attempt to capture the "degree" of democracy in a given country. Regardless of the specification, each of these variables is censured and, when used in regressions, typically normalized to fall in the interval $[0, 1]$.

regressions, both income levels and income growth are strong predictors of positive changes in democracy. They find that human capital levels – particularly when measured via primary school attainment – are an even stronger predictor of democratization than income variables.

On the theoretical side, Benhabib and Rustichini (1996) and Benahabib and Przeworski (2004) are the most closely related to my work. The former paper studies an infinite horizon growth model in which heterogeneous agents play a sequence of Nash bargaining games to determine the surplus division in each period. Agents have diminishing marginal utilities of consumption and the production technology exhibits decreasing returns to scale in the accumulated factor, and the degree of redistributive conflict in each period is determined by the relative marginal gains of present consumption via expropriation and future consumption through saving and accumulation. The authors show that, when the marginal utility of consumption is rapidly diminishing, poorer countries grow more slowly and may even become caught in growth traps. On the other hand, when the curvature of the utility function is sufficiently less than the curvature of the production function, the incentive constraints on conflict bind at high wealth levels. The results of the latter paper are similar: agents are subject to an income-invariant cost of repression in autocratic regimes, and democratization occurs when this cost outweighs the marginal utility gain from consumption. Hence, there exists a critical level of capital (i.e., wealth) above which autocracy cannot be sustained.

This approach gives results consistent with the original modernization hypothesis, since the *level* of income is the key determinant of political conflict and democratization.[7] In contrast, the present paper abstracts from level effects entirely by assuming utility functions that are linear in consumption and a linear production technology. Hence, my results do not rely on specific assumptions about the concavity of the utility function and a neoclassical production function. This results both in greater tractability[8] and closer accordance with models of endogenous growth that do not have diminishing returns in the long run.

Inequality, the demand for redistribution, and its effect on growth: Two related literatures examine the determinants of the demand for redistributive policies and

[7]Since these models are dynamic and (history dependent) SPNE are allowed, growth rates play an indirect role by altering the future set of feasible redistributions. However, the level of capital accumulation is still the primary mechanism.

[8]I also restrict attention to Markovian strategies, which also adds to the model's tractability, if at some cost of generality.

the effects of these policies – in particular, distortionary taxation – on growth outcomes. Since the demand for redistribution is closely linked to the degree of economic inequality, this line of work therefore also indirectly addresses the question of the effect of inequality on growth.

Much of economists' thinking about the demand for redistribution is based on political economy models that exploit the median voter theorem, and hence correspond to a "one-person-one-vote" ideal. A central question in this class of models is therefore why we should observe the wide range of redistributive institutions that exist today in seemingly similar developed countries. In a seminal paper, Piketty (1995) shows how heterogeneous priors and private learning about the economic return to effort can lead to multiple equilibrium levels of redistribution. Agents live for a single period in which they may undergo economic mobility, and at the end of which they vote on the subsequent period's level of redistributive taxation with the objective of maximizing welfare for "poor children" in the next generation. Learning about the determinants of mobility determine how agents vote at the end of their lives. [9] This result is interpreted as a "Bayesian learning" interpretation for the very different redistributive policies instituted in, e.g., the United States and many Scandinavian countries.

Benabou and Ok (2001) address a distinct but related question: under what circumstances will voters rationally choose low levels of redistribution when the income distribution is right-skewed? The key insight to emerge from this paper is that, if there is persistence in taxation policies, even agents with below-median income will vote for zero redistribution if the "transition function" governing mobility prospects is sufficiently concave.[10] In simple terms, poor agents dislike future redistribution if their prospects for upward mobility are good enough.

Answers to the second question – how redistribution affects growth – come in two classes, both of which typically rely on two mechanisms. The first class of models assumes an orderly – though not necessarily egalitarian – political process, so that the median voter theorem may be invoked. The seminal paper of Alesina and Rodrik (1994) develops a model of endogenous growth to show that the rate of growth follows an inverse U-shaped curve as a function of redistributive taxation. This non-monotonicity is driven by their assumption

[9] Unlike the papers cited in footnote 5, agents in this model are fully Bayesian. The multiplicity of ergodic belief distributions is driven by the non-observability of others' priors and variations in idiosyncratic income shocks. Alesina and Angeletos (2005) develop a related model in which beliefs about fairness drive equilibrium multiplicity.

[10] Mathematically, this is just a corollary of Jensen's Inequality.

that taxation funds productive government services. Their key result, however, is that increased inequality – measured as the difference between average and median incomes – monotonically decreases the growth rate. As shown in Benabou (1996) and related papers, this result can be reversed when another channel is taken into account. If the marginal return on investment varies with income and credit markets are imperfect, positive rates of redistribution increase allocative efficiency and equilibrium growth rates.[11] When both effects are incorporated in a single model, the rate of redistribution has a non-monotonic effect on growth rates. Which effect wins out is a function of model parameters – namely, the location of the pivotal agent in the wealth distribution[12] and the degree to which the marginal return of investment varies with wealth.

Empirically, the non-monotonic effect of inequality and redistribution on growth suggested by Benabou's analysis seems to be closer to reality. While some degree of redistribution seems to be necessary for growth – particularly at early stages of development when poor agents may not even have access to, e.g., primary education – the data shows fairly consistently that too much redistribution is bad for growth.

The second class of models posits that inequality effects growth through political and economic conflict.[13] Benabou (1996) analyzes a "repeated" prisoner's dilemma with capital accumulation. Due to the same consumption-savings tradeoff as in Benhabib and Rustichini (1996), there is a maximal growth rate that can be sustained in any SPNE. Conflict over surplus allocation becomes more likely – and the maximal sustainable growth rate lower – when agents' claims to surplus differ greatly from their outside options and their gains from expropriation. This effect appears in my model when the payoffs to revolution are "state-dependent;" see Section 4. These insights are also similar to those stemming from the model of Acemoglu and Robinson (2006), in which conflict becomes more likely as the gap between *de jure* and *de facto* political power becomes larger.

[11] This can be modeled in several ways. Many papers, including the political economy models of Acemoglu and Robinson (2000, 2002), follow Galor and Zeira (1993) in assuming that there are non-convexities in the accumulation technology. In particular, a minimum amount of investment is required to induce a nontrivial rate of human capital accumulation. Benabou (1996) assumes that agents undertake production individually, and that the production function has decreasing returns to scale. In the former specification, redistribution allows poor agents to begin accumulation; in the former, it exploits differences in poor and rich agents' marginal returns to (human) capital in production.

[12] It is easy to depart from the median voter benchmark and allow for variable degrees of "wealth bias" in the political process. Benabou (1996) and related works present a tractable scheme for doing this with a continuous wealth distribution. I follow Acemoglu and Robinson (2000, 2002, 2006) in assuming a discrete distribution.

[13] Benhabib and Rustichini (1996) and Benahabib and Przeworski (2004) fall in this category, though they differ from the models discussed here in that they rely on level effects.

Growth shocks, civil conflict, and property rights: While the models of inequality and conflict described above highlight the role of property rights in protecting against expropriation and hence encouraging investment, they do so in perfect information settings. A third literature explores the implications of stochastic fluctuations in growth for sociopolitical conflict and, indirectly, for growth.

Rodrik (1999) empirically investigates the role of negative "growth shocks" on subsequent growth outcomes in the postwar period. In particular, he emphasizes that a sizable fraction of the growth divergence between East Asia and the rest of the developing world since 1960 is due to growth collapses in these other regions since the mid-1970s. He attributes this to weaknesses in their conflict-management institutions – i.e., property rights and the ability to reach compromise at the legislative level when instituting macroeconomic stabilization policies. In particular, and consistent with Friedman's (2006) theory, he posits that negative growth shocks exacerbate latent social tensions and can result in on-path conflict. Consistent with these predictions, he finds that institutional quality and democracy act as buffers against growth shocks, and that income inequality and ethnolinguisitic fractionalization make conflict more likely and contribute to persistent negative effects of transient growth shocks. Importantly, he finds that ethno-linguisitic fractionalization is a somewhat more robust predictor of conflict than income inequality.

While the conflicts envisioned by Rodrik and others in political economy are "peaceful," development economists are interested in settings that are conducive to the onset of outright civil war. A popular explanation has been the lower opportunity cost of war induced by negative growth shocks. Using rainfall as an instrument for agricultural income, Miguel et al (2004) show that negative income shocks are good predictors of civil conflict in sub-Saharan Africa. Dal Bo and Dal Bo (2008) and Chassang and Padro i Miquel (2009) develop theoretical models that find support for the opportunity cost hypothesis. These models seem similar to mine at first glance, but the mechanisms at work and the interpretations are very different. In those papers, the opportunity cost to engage in civil conflict is lower in bad economic times because the one-period loss from not undertaking production is lower. In other words, they describe settings in which the tradeoff is between "staying on the farm" and "taking up arms" for a set period of time. Everything in my model (at least the dynamic version in Sections 3-5) is driven by expectations about long-run effects and trends.

2 A simple example: the role of growth and redistribution

Consider an economy that runs in discrete time $t \in \mathbb{N}$, which in each period is populated by a continuum of agents with mass one. Agents live for a single period and are of two types: rich (r) and poor (p).[14] The population of rich agents has measure $\delta < \frac{1}{2}$ so that the median voter is poor. For simplicity, all agents of a given type are identical, implying that their preferred actions are identical and hence that each type can be modeled as a single agent.[15] I also abstract from the consumption/saving decision: agents have utility over lifetime wealth, which is passed on in full to their offspring. Formally, the utility of a type i agent with wealth y is $u^i(y) = y$. In period $t = 0$, let the output of the economy be normalized to one so that the initial per capita wealth of rich agents is $y_0^r = \frac{\theta_0}{\delta}$, and that of poor agents $y_0^p = \frac{1-\theta_0}{1-\delta}$, where I assume that $\theta_0 > \delta$.

Suppose that aggregate wealth in this economy grows at the known constant rate $\phi \geq 0$ in each period, i.e., aggregate wealth follows the law of motion $y_{t+1} = (1+\phi)y_t$. I define the *aggregate surplus in period* t to be $S_t := y_t - y_{t-1} = \phi \cdot y_{t-1}$. Rich agents capture fraction $\alpha_t \in [0, 1]$ of the aggregate surplus, leaving fraction $1 - \alpha_t$ for poor agents. Hence, the per capita wealth for each type follow the laws of motion

$$y_{t+1}^r = \frac{\theta_t + \phi\alpha_t}{\delta} y_t = y_t^r + \frac{\phi\alpha_t}{\delta} y_t \tag{1}$$

and

$$y_{t+1}^p = \frac{(1-\theta_t) + \phi(1-\alpha_t)}{1-\delta} y_t = y_t^p + \frac{\phi(1-\alpha_t)}{1-\delta} y_t, \tag{2}$$

so that the income share going to the rich evolves as

$$\theta_{t+1} = \frac{\theta_t + \phi\alpha_t}{1+\phi}, \tag{3}$$

which is constant if and only if $\alpha_t = \theta_t$.

Consider the following stage game played during each period $t \in \mathbb{N}$ among generation-t agents. Elites control the political process and modes of production and therefore set $\alpha_t \in [0, 1]$ in each period. I abstract from the hold-up problem, so that even if rich agents

[14]Following Acemoglu and Robinson (2006), I refer to the former group interchangeably as "rich agents" or "elites."

[15]Formally, this also assumes that agents within each group have solved their collective action problem and are able to coordinate on the same action.

move first they can commit to this level of surplus division.[16] Once α_t is announced, poor agents choose whether to accept the offer and allow production to proceed, or to stage a revolution. In the case of revolt, a fraction $\mu \in (0,1)$ of the economy's period t output is destroyed, and poor agents take permanent control of the remaining wealth. After a revolution in period $t+1$, which I denote by $R(t)$,[17] wealth evolves as

$$y_s^p(R(t)) = \frac{1-\mu}{1-\delta} \cdot (1+\phi) y_t$$

and

$$y_s^r(R(t)) = 0$$

for all $s \geq t+1$. The interpretation is that, after a revolution, poor agents divide the total stock of wealth evenly among each other and rich agents are left with nothing.[18] These allocations persist for all future periods. I now characterize the (essentially) unique equilibrium of this stage game.

Lemma 1: *There exists an essentially unique subgame perfect Nash equilibrium of this stage game. Given a level of inequality θ_{t-1} from the previous period, the equilibrium level of surplus division is $\alpha_t^* = \frac{(1+\phi)\mu - \theta_{t-1}}{\phi} \geq 0$ if feasible. Revolution occurs if and only if the non-negativity constraint does not bind.*

Proof. See the Appendix. □

This lemma leads immediately to the following result.

[16] If elites chose α and poor agents chose whether to revolt simultaneously, there would (for large enough ϕ) be multiple Nash equilibria, since the normal form game would have a coordination structure. One class of equilibria would have low redistribution (which only needs to be below some critical threshold) and revolt; the other class would consist of a single equilibrium with high redistribution and no revolt. In the extensive form game described here, I can effectively select either of these classes through the timing of moves. In particular, if elites move first and have commitment power, they will optimally choose to redistribute to the point where poor agents do not revolt; revolt will not occur by sequential rationality. If poor agents move first, they will revolt unless it is sequentially rational for elites to redistribute after the revolution threat has subsided. Since in this example one-period lives imply that elites will never have an incentive to redistribute ex-post, I let them move first.

[17] Throughout, I use upper case letters to refer to states and actions. Lower case letters are reserved to index agent types. This is to avoid confusion between R (revolt) and r (rich agent or elite).

[18] That elites get exactly zero wealth after revolution is not important. All that is needed for the results of this paper to hold is that elites strictly prefer redistribution to revolution.

Proposition 1: *Given any initial level of inequality $\theta_0 \leq \mu$ and for any $\phi \geq 0$, revolution never occurs and $\theta_t = \mu\ \forall t \in \mathbb{N}$. Given any initial level of inequality $\theta_0 > \mu$, there is never revolution and $\theta_t = \mu\ \forall t \in \mathbb{N}$ if and only if $\phi \geq \bar{\phi}$, where $\bar{\phi}$ solves $(1+\bar{\phi}) = \frac{\theta_0}{\mu}$. For any $\phi \in [0, \bar{\phi})$, revolution in the first period is unavoidable.*

Proof. See the Appendix. □

This proposition is the simplest possible way of capturing the fact that economic growth – and particularly the prospect that this growth will enrich all members of society – can mitigate social and political conflict. Even for high initial levels of inequality and low costs of revolt,[19] the coupling of rapid growth and frictionless redistributive channels allows for social tensions to remain latent, with revolution never occurring on the equilibrium path. The intuition is as follows. Revolution can be avoided if and only if there exists a feasible transfer that induces $y_t^p \geq y_t^p(R(t))$ (which is equivalent to the condition in Lemma 1). When inequality is high ($\theta > \mu$), such a transfer needs to be large for this incentive constraint to hold. But the size of the maximal feasible transfer is increasing in the growth rate, because *higher rates of growth increase S_t, the period-t aggregate surplus.*

The remainder of this paper is devoted to extending this insight to more realistic settings. First, agents in this example only live for one period. The results obtained therefore neglect the effects of growth compounded over many periods. When agents optimize over longer time horizons, the sensitivity of growth to the revolution decision plays an important role in poor agents' incentives to revolt. Second, the growth rate in this example is constant in time. In reality, people's decisions are heavily influenced by their expectations about their future economic prospects. When the economic environment is weak, there is often uncertainty about how long it will take to recover to a period of robust growth. Hence, it would be desirable for a more complete model to include this kind of uncertainty. Third, the surplus division technology is completely frictionless in this example. In reality, transfers often cannot be completely directed. More importantly, political institutions that give the elites full *de jure* political power inherently lead to commitment problems. As Acemoglu and Robinson (2000, 2002, 2006) have emphasized, the ability to commit to future redistribution provides a powerful rationale for elites to voluntary establish democracy in some cases. I study how economic conditions mitigate or exacerbate these commitment issues. Fourth, this example assumed that all agents in the economy accumulate wealth at the same rate, which is not empirically accurate (Benabou (1996), Galor and Zeira (1993),

[19] See the discussion in Section 4 for more on the interpretation of these costs.

Piketty (2014)). Introducing variable accumulation rates lets me study how inequality and social tensions evolve along the development path. In the subsequent sections, I take up each of these extensions in turn, showing that the main insight generated by this simple example – that economic growth can mitigate conflict – continues to hold in more realistic settings.

3 The dynamic model

Consider an infinite horizon variant of the model from Section 2, where now agents live for all $t \in \mathbb{N}$ and discount the future at rate $\beta \in (0,1)$. Agents are risk neutral so that expected lifetime utility starting in period t for an agent of type $i \in \{r, p\}$ is

$$V_t^i = \mathbb{E}\Big[\sum_{s=t}^{\infty} \beta^{s-t} h_s^i \Big| \mathcal{F}_t\Big], \tag{4}$$

where $\mathcal{F}_t$ is the public history at time t and h_s^i is type i's period s wealth. The public history contains the entire sequence of previous and present aggregate states (described below) and all past actions by elites and poor agents. Formally, $\mathcal{F}_t = s_t \cup \{(s_r, \vec{a}_r)\}_{r=0}^{t-1}$, where $\vec{a}_t$ is the action profile realized in period t.[20]

In each period, the economy is in one of two aggregate states – high (H) or low (L) – and evolves according to the following Markov process. Given knowledge of the current state, the conditional probability that the economy will be in the high aggregate state in the subsequent period is

$$\Pr(s_{t+1} = H | s_t) = \begin{cases} q_H, & \text{if } s_t = H \\ 1 - q_L, & \text{if } s_t = L \end{cases}$$

and the economy transitions to the low state with complementary probabilities

[20]Throughout the paper, I abstract from the consumption/saving decision. It would be straightforward to introduce an exogenous saving rate and define utility over consumption streams. Value functions would then be concave in the savings rate (Benabou (1996)). However, this extension does not add much to the present analysis – all statements made here about the growth rate would simply be translated into statements about the product of the growth rate and savings rate. Modeling agents as Ramsey consumers and solving for their optimal consumption plans would significantly complicate the analysis, and I do not attempt to do so here. See, e.g., Benhabib and Rustichini (1996) for a suggestive example.

$$\Pr(s_{t+1} = L|s_t) = \begin{cases} 1 - q_H, & \text{if } s_t = H \\ q_L, & \text{if } s_t = L \end{cases}$$

so that q_{s_t} gives the persistence of state $s_t \in \{L, H\}$. Expectations are taken with respect to the distribution induced by this process.[21]

While the focus of the paper is on an economy with aggregate growth, it is easiest to begin by analyzing an analogous endowment economy without accumulation. This allows me to obtain clean closed-form solutions; I then show that the same qualitative features hold in the economy with growth. In the endowment economy, agents are endowed each period with type-specific wealth h^i, $i \in \{p, r\}$, which fully depreciates before the start of the next period.[22] As in Section 2, these can be written as $h^r = \frac{\theta}{\delta}$ and $h^p = \frac{1-\theta}{1-\delta}$, with $\theta > \delta$. Let $H = \delta h^r + (1-\delta)h^p$ be the total amount of capital in the economy. The interpretation is now that h^i is wealth is measured in ownership of a productive asset, which I call "capital" for concreteness.[23] Assume that capital is the only input in production, which is carried out individually.

There are two sectors in which production can take place, one formal and one informal, where the former is more productive. In particular, the marginal product of capital in the informal sector is $B \geq 1$ in both states, and the marginal product in the formal sector is state-dependent and given by $A_H = (1+\phi)B$ and $A_L = B$, where $\phi > 0$.[24] Assume further that the informal sector is not taxable, but that both sectors are vulnerable to other types of expropriation. The maximum tax rate in each state is given by $\hat{\tau}_H = \frac{\phi}{1+\phi}$ and $\hat{\tau}_L = 0$, so that in "boom" times greater redistribution is possible.[25] Tax revenues are rebated lump sum to the entire population. Denote these rebates by T_t. I assume that the government must balance the budget in each period, so that $T_t = \int_{F_t} \tau_t h_j dj$, where $j \in F_t$ denotes that

[21] Refer to the proof of Proposition 5 for a more formal definition.

[22] In this case, post-production wealth can be interpreted as consumption levels without considering any savings choice.

[23] As Acemoglu and Robinson (2000, 2002) note, this may be best thought of as a composition of physical and human capital. This broader interpretation is consistent with the literature on growth with incomplete capital markets (e.g., Benabou (1996); Galor and Zeira (1993)) and the recent findings of Murten and Wacziarg (2013) on the education-democracy correlation.

[24] The normalization $A_L = B$ and hence $\hat{\tau}_L = 0$ is for convenience. If $A_L < B$, all production would move to the informal sector in any case. Qualitative results would not change if $A_H > A_L > B$, but the calculations become messier.

[25] These maximal rates solve $(1 - \tau_{s_t})A_{s_t} = B$. If $\tau_t > \hat{\tau}_{s_t}$, all production moves to the informal sector and tax revenues are zero, which can never be optimal from a redistributive perspective.

agent $j \in [0,1]$ has his capital committed to the formal sector in period t. For $\tau_t \leq \hat{\tau}_{s_t}$, $F_t = [0,1]$ (up to sets of measure zero) so total revenues (which are equivalent to per capita rebates with the population normalized to size one) are $T_t = \tau_t \cdot H$.

I emphasize that the distribution technology need not – and probably should not – be literally interpreted as taxation on income or capital gains. It is simply a way of allocating economic gains between the two groups that is controlled by the group with political power. I use a flat rate tax because it facilitates direct comparison with the models of Acemoglu and Robinson (2000, 2002, 2006).[26] The restriction to non-targeted transfers is also technically convenient, as it reduces the dimensionality of the policy space and lets me invoke the Median Voter Theorem. It is worth collecting some observations about the technical properties of this technology.

Remark 1: *Let (τ_{s_t}, T_{s_t}) be as described above. Then the redistribution technology has the following properties.*

1. *When $s_t = H$ and there is no accumulation, the minimum post-tax wealth left for rich agents (before rebates) is $h_t^{r,min} = B \cdot h_t^r$. When there is accumulation and $B = 1$, this quantity becomes $h_t^{r,min} = h_{t-1}^r$. That is, the instruments allow only for income- (as opposed to wealth-) redistribution.*

2. *When there is accumulation, in the notation of Section 2, the smallest income share that goes to the elites is $\alpha_t^{min} > 0$. This follows from the fact that transfers are not targeted. When there is no accumulation, an analogous result obtains if I redefine the aggregate surplus as $S_t' = H_t(H) - H_t(L)$, where $H_t(s_t)$ is aggregate output in state s_t.*

3. *Because I abstract from intertemporal optimization over consumption and savings, taxes do not distort investment decisions.*

The first observation preserves the structure of the technology from Section 2 in that *only (a fraction of) current period output can be redistributed.* One can clearly envision transfer schemes in which previously-accumulated wealth is redistributed. A prime example of this type of scheme is the land reforms that took place in post-war Japan (see, e.g., Alesina and Rodrik (1994)). These kinds of transfers typically occur either (1) during a violent

[26]Refer to the commentary in the next section for further interpretation.

revolution or regime change or (2) as a product of dramatic nation-wide reforms. The latter instance corresponds to the case of post-war Japan. In this paper, I restrict attention to the first instance.[27] The latter two observations are self-evident.

In each period (assuming revolution has not yet occurred), the timeline is as follows:

1. The current state $s_t \in \mathcal{F}_t$ is revealed.

2. The current tax rate τ_t is set, after which capital is committed to production.

3. Poor agents decide whether to initiate a revolution, which, if undertaken, succeeds with probability one.

4. Returns to production are realized and distributed accordingly.

With this framework in place, I proceed to define and characterize equilibria in this economy.

4 Equilibrium characterization

I restrict attention throughout to pure strategy Markov perfect equilibria, which henceforth are simply referred to as "equilibria." This simplification drastically reduces the equilibrium set relative to subgame perfect Nash equilibrium by eliminating history dependence. Indeed, for a given set of parameters and sequence of realized states $\mathbf{s} = \{s_t\}_{t=0}^{\infty}$, there will always be an essentially unique equilibrium in this economy. The stationarity of Markovian equilibria also allows me to utilize elementary dynamic programming arguments to characterize this equilibrium throughout.[28] Formally, equilibria are classified by the group in political power and are defined as follows.

Definition 1: *A (pure strategy Markov perfect) equilibrium (under elite control) is a pair of mappings $\sigma^p : \{s_t, \theta_t, H_t, \tau_t\} \to \{revolt, \ no \ revolt\}$ and $\sigma^r : \{s_t, \theta_t, H_t\} \to \{\tau \in [0, \hat{\tau}_{s_t}], \Delta\}$ so that $\forall t \in \mathbb{N}$, $V_t^p(\sigma^p, \sigma^r) \geq V_t^p(\tilde{\sigma}^p, \sigma^r)$ and $V_t^r(\sigma^p, \sigma^r) \geq V_t^r(\sigma^p, \tilde{\sigma}^r)$ for all alternative*

[27] This is largely a matter of interpretation. While I do not consider the problem of a government choosing among various types of institutional reform, the payoffs under revolution can just as well be interpreted as the payoffs after a set of redistributive reforms. In this case, $\mu \in (0, 1)$ would represent the deadweight loss due to wealth reallocation.

[28] Acemoglu and Robinson (2000, 2002, 2006) emphasize the appeal of the Markovian restriction for its natural representation of commitment problems in political economy. This interpretation becomes important when I discuss incentives for democratization later.

mappings $\tilde{\sigma}^i,\ i \in \{p, r\}$.

The payoff-relevant state variables in this economy are (1) the current aggregate state s_t, (2) the level of wealth inequality θ_t, and (3) the aggregate wealth level H_t. When elites control the political process, their strategy space consists of all (measurable) mappings from the vector of state variables to a feasible tax rate τ and a democratization decision Δ. Without loss of generality, let $\Delta = 1$ denote democratization and $\Delta = 0$ denote that elites hold onto political power. Strategies for poor agents are similarly (measurable) mappings from the vector of state variables and, assuming $\Delta = 0$, the announced tax rate. If $\Delta = 1$ in any period $s \leq t$, the strategies are defined as follows.

Definition 2: *A (pure strategy Markov perfect) equilibrium (under democracy) is a pair of mappings $\sigma^p : \{s_t, \theta_t, H_t\} \to \tau \in [0, \hat{\tau}_{s_t}]$ and $\sigma^r : \{s_t, \theta_t, H_t\} \to \emptyset$ so that $\forall t \in \mathbb{N}$, $V_t^p(\sigma^p, \sigma^r) \geq V_t^p(\tilde{\sigma}^p, \sigma^r)$ and $V_t^r(\sigma^p, \sigma^r) \geq V_t^r(\sigma^p, \tilde{\sigma}^r)$ for all alternative mappings $\tilde{\sigma}^i,\ i \in \{p, r\}$.*

In democracy, the poor agents hold political power and set their optimal tax rate. Elites, out of political power, are restricted to null actions.

The strategy spaces are simple, and optimal play can be determined by a small set of incentive constraints. Since these incentive constraints are more intuitive than statements about strategies, the remainder of my results will only reference the constraints. The formal translations are obvious. In particular, I need the following definitions.

Definition 3: *I say that "revolution occurs in state $s_t \in \{L, H\}$ at time t" if it is strictly optimal for poor agents to stage revolution in response to any feasible sequence of transfers $\{\tau_s\}_{s=t}^{\infty}$. Denote this event by $R(s_t, t)$.*

Definition 4: *I say that "the revolution constraint binds in state $s_t \in \{L, H\}$ at time t" if it is strictly optimal for poor agents to stage revolution, conditional on all present and future transfers being exactly zero: $\tau_s = 0\ \forall s \geq t$.*

Definition 5: *I say that "democratization occurs in state $s_t \in \{L, H\}$ at time t" if it is strictly optimal for elites to democratize, conditional on optimal response by the poor agents.*

Once I have characterized the appropriate value functions, it is easy to see how these three definitions translate directly into incentive constraints (inequalities between the value functions) and therefore equilibrium strategies.

The following fact should be clear just from the definitions, but I state it here so that there is no ambiguity going forward. The first part is trivial; the second follows from the Median Voter Theorem and the fact that the policy space is one-dimensional.

Lemma 3: *Under elite control, $\tau_t = 0$ unless the revolution constraint binds at (s_t, t). Under democracy, $\tau_t = \hat{\tau}_{s_t}$ $\forall t \in \mathbb{N}$.*

The analysis proceeds in two stages. First, I consider the endowment economy without growth described in Section 3. This allows for a sharp, time-independent characterization of the relevant constraints. Second, I add accumulation to the model in the natural way. In this context, I discuss ergodic properties of the economy, inequality dynamics, and the transition to and stability of democracy.

4.1 An economy without accumulation

Without accumulation, income shares of the rich and poor are time invariant. Therefore, revolution occurs (in any period t) if and only if it occurs immediately. Moreover, since I have assumed that elites receive payoffs of zero under revolution, it is always optimal to fend off revolution through maximally redistributive taxes, if feasible.

Let $\hat{V}^p_{s_t}$ denote the value function for poor agents in state $s_t \in \{L, H\}$ when there is maximal redistribution. These value functions will be used to determine whether revolution can be avoided through transfers. It is clear that these value functions must satisfy the recursive equations

$$\hat{V}^p_H = (1 - \hat{\tau}_H) A_H h^p + \hat{\tau}_H A_H H + \beta \Big[q_H \hat{V}^p_H + (1 - q_H) \hat{V}^p_L \Big] \tag{5}$$

and

$$\hat{V}^p_L = B h^p + \beta \Big[q_L \hat{V}^p_L + (1 - q_L) \hat{V}^p_H \Big]. \tag{6}$$

Note that, once the definitions of A_H and $\hat{\tau}_H$ are substituted in to Equation (5), the only term that explicitly depends on ϕ is the middle term $\hat{\tau}_H A_H H = \phi BH$. This is explained as follows. Each agent taxed to the point where his productivity is B, which is the same as in the low state. In the high state, however, there are positive tax revenues that increase with ϕ. This is where one needs to be somewhat careful with the strict interpretation that redistribution works through taxation and government transfers. In many macroeconomic models, the optimal tax scheme is procyclical, which is the case here by virtue of the participation constraint $(1-\tau)A_H \geq B$. But a large class of macroeconomic models also suggests that deficit spending (fiscal stimulus) is optimal during recessions, which I have explicitly ruled out of the model by assuming that the government has a perfectly balanced budget in each period. For this reason, it is more palatable to abstract from government as a standalone third party and hence from taxation as described above. But the abstract surplus division scheme from Section 2 illustrates that the effects of growth on the revolution decision has nothing *a priori* to do with taxes. The only important features are: (1) there is more surplus to be divided in the high state, and (2) redistribution is more difficult in the low state due to frictions inherent in the surplus division technology.

Equations (5) and (6) can be solved together to get[29]

$$\hat{V}_H^p = \frac{BH}{1-\beta}\Big[\frac{\phi(1-\beta q_L)}{1+\beta(1-q_L-q_H)} + \frac{1-\theta}{1-\delta}\Big] \tag{7}$$

and

$$\hat{V}_L^p = \frac{BH}{1-\beta}\Big[\frac{\phi\beta(1-q_L)}{1+\beta(1-q_L-q_H)} + \frac{1-\theta}{1-\delta}\Big] \tag{8}$$

so that the difference in expected lifetime utility between states $\Delta\hat{V}^P := \hat{V}_H^P - \hat{V}_L^p$ is given by

$$\Delta\hat{V}^p = \frac{BH}{1-\beta}\cdot\frac{\phi(1-\beta)}{1+\beta(1-q_L-q_H)} > 0. \tag{9}$$

Observe that this difference does not depend on the inequality parameter θ but is strictly increasing in both persistence probabilities q_H and q_L. In particular, both value functions are strictly decreasing in q_L, but $\hat{V}_L^P$ decreases faster. Similarly, both value functions are strictly increasing in q_H and $\hat{V}_H^P$ increases faster.

Similarly, let $V^p(s_t, h^p, \tau \equiv 0)$ denote the poor agents' value functions when there is zero redistribution in the present and in all future periods. These will be used in characterizing

[29]See the Appendix for proof.

the revolution constraint. These must satisfy the recursive equations

$$V^p(H, h^p, \tau \equiv 0) = (1+\phi) \cdot B \cdot h^p + \beta q_H V^p(H, h^p, \tau \equiv 0) + \beta(1-q_H) V^p(L, h^p, \tau \equiv 0) \quad (10)$$

and

$$V^p(L, h^p, \tau \equiv 0) = B \cdot h^p + \beta q_L V^p(L, h^p, \tau \equiv 0) + \beta(1-q_L) V^p(H, h^p, \tau \equiv 0). \quad (11)$$

Solving (10) and (11) together yields

$$V^p(H, h^p, \tau \equiv 0) = \frac{BH}{1-\beta} \cdot \frac{1-\theta}{1-\delta} \cdot \frac{1+\phi+\beta(1-(1+\phi)q_L - q_H)}{1+\beta(1-q_L-q_H)} \quad (12)$$

and

$$V^p(L, h^p, \tau \equiv 0) = \frac{BH}{1-\beta} \cdot \frac{1-\theta}{1-\delta} \cdot \frac{1+\beta((1-q_L)(1+\phi)-q_H)}{1+\beta(1-q_L-q_H)}, \quad (13)$$

and it is easily verified that $V^p(H, h^p, \tau \equiv 0) > V^p(L, h^p, \tau \equiv 0)$. However, an ordering on $\hat{V}_L^p$ and $V^p(H, h^p, \tau \equiv 0)$ is not completely immediate. Straightforward algebra yields that

$$\hat{V}_L^p > V^p(H, h^p, \tau \equiv 0) \iff \frac{\beta(1-q_L)}{1-\beta q_L} > \frac{1-\theta}{1-\delta}. \quad (14)$$

This is summarized in the following lemma.

Lemma 4: *If inequality is sufficiently high ($\theta \approx 1$) or agents are sufficiently patient ($\beta \approx 1$), maximal redistribution $\forall t$ starting in the low state is better for poor agents than zero redistribution $\forall t$ starting in the high state. This never holds when the low state is very persistent ($q_L \approx 1$).*

Proof. Immediate from Equation (14). □

This result is intuitive. When agents are very patient, they internalize the gains from redistribution in the high state, even if it is in the distant future. Similarly, when the low state is perfectly persistent, the high state is never again realized, so the former effect is nullified. When inequality is very high, it is feasible to have relatively large transfers in the future, since the proportion of the tax base comprised of rich agents is large. Which way the inequality runs does not have an appreciable effect on equilibrium play.

I can then restate Definitions 3 and 4 as follows.

Definition 3': *Revolution occurs in state* $s_t \in \{L, H\}$ *at time* t *iff* $V^P(R(s_t, t)) > \hat{V}^p_{s_t}$.

Definition 4': *The revolution constraint binds in state* $s_t \in \{L, H\}$ *at time* t *iff* $V^P(R(s_t, t)) > V^p(H, h^p, \tau \equiv 0)$.

Below, I consider the implications of two different assumptions about the productivity implications of revolution, each of which captures different aspects of the revolution decision. Although I use the first case for the remainder of the paper (this is formally stated in Assumption 1 below), it is useful to understand when alternative results are obtained, their connection with the extant literature, and arguments against their empirical relevance.

4.1.1 Case 1 (State-Independent Constraints):

Assume that, once revolution takes place, the economy never again enters the high productivity state. The value functions in this case are given by

$$V^p(R) = \frac{1-\mu}{1-\delta} \cdot \frac{BH}{1-\beta} \tag{15}$$

and

$$V^r(R) = 0 \tag{16}$$

independently of the current state.[30] Note that these are analogous to the payoffs under revolution in Section 2, but in perpetuity.

The following proposition characterizes when revolution is imminent. For concision, I say that an event is "more likely" if it holds for a larger set of parameter values.

Proposition 2: *When the payoffs to revolution are state-independent,*

1. $R(L) \iff \frac{\theta-\mu}{1-\delta} > \frac{\phi\beta(1-q_L)}{1+\beta(1-q_L-q_H)}$ *and* $R(H) \iff \frac{\theta-\mu}{1-\delta} > \frac{\phi(1-\beta q_L)}{1+\beta(1-q_L-q_H)}$.
2. *For all parameter values* $R(H) \implies R(L)$, *i.e., revolution in the low state is always more likely.*
3. $R(L)$ *and* $R(H)$ *both become less likely as:* ϕ *increases,* q_H *increases, and* q_L *decreases.*

[30] I suppress the time- and state-dependence in the revolution term when it does not cause confusion.

4. *$R(L)$ becomes less likely as β increases, while $R(H)$ becomes more likely.*

Proof. See the Appendix. □

This result extends the intuitions from Proposition 1 to the present dynamic environment. As in Section 2, even for high inequality ($\theta > \mu$), there exists a critical productivity level $\phi^* > 0$ such that revolution is avoided in both states for all $\phi \geq \phi^*$. Moreover, revolution is always more likely in the low state, as poor agents' prospects of benefitting from growth are delayed at least one period. Similarly, the expectation of sustained growth (a high q_H) makes revolution less likely in both states, while the prospect of a prolonged downturn (a high q_L) increases the likelihood of revolution. The comparative statics with respect to the discount factor are also intuitively appealing. When in the low state, increased patience allows agents to look forward to good times in the future, and these positive prospects work to mitigate the revolution threat. In contrast, in the high state patient agents internalize the transience of the present windfall – i.e., hard times in the future loom larger. In the limit of perfect patience $\beta \to 1$, the constraints are equivalent.

It is also useful to think about the implications of state persistence for the incidence of revolution.

Corollary 1: *Let $q_H = q_L = q$. Then $R(L)$ becomes more likely and the $R(H)$ becomes less likely as q increases.*

Proof. See the Appendix. □

This confirms the basic intuition that increased persistence is good in the high state and bad in the low state. In particular, since this comparative static exercise varies both persistence probabilities in the same way, higher persistence means that the economy is more "locked in" to the current state. To sharpen this characterization, it is useful to define the following quantities.

Definition 5: $\phi_s^* := \inf\{\phi \in \mathbb{R}_+ |$ *not* $R(s)\ \forall(\mu, \theta) \in [0,1]^2\}$ *for* $s \in \{L, H\}$.[31]

Given the incentive constraints for the events $R(s)$ in Proposition 2, it clearly suffices to find the numbers ϕ_s^* so that each of the constraints binds as an equality for $(\mu, \theta) = (0, 1)$. This

[31] I do not specify whether the events $R(s)$ correspond to state-dependent or state-independent constraints whenever the context is clear.

cost-inequality pair is the most extreme one feasible: revolution is costless for poor agents, and rich agents hold all of the economy's wealth. In this case, poor agents' only source of income is lump-sum transfers from the elites. Because the volume of redistribution in a given period depends on the productivity differential between the low and high state ϕ, this differential must be large enough to satisfy the poor agents' incentive constraints. The following proposition characterizes the dependence of the ϕ_s^* on the persistence probabilities q_s.

Proposition 3: *As $q_L \to 1$, $\phi_L^* \to \infty$ and $\phi_H^* \to \frac{1-\beta q_H}{(1-\delta)(1-\beta)} > \frac{1}{1-\delta}$. As $q_H \to 1$, $\phi_L^* \to \frac{1-\beta q_L}{\beta(1-q_L)(1-\beta)} > \frac{1}{1-\delta}$ and $\phi_H^* \to \frac{1}{1-\delta} > 1$.*

Proof. See the Appendix. □

These qualitative results are appealing. In the case that the low state is arbitrarily persistent ($q_L \to 1$), no amount of productivity in the high state (which will never be realized) can ward off revolution for all cost-inequality pairs. Indeed, when $q_L = 1$, $R(L)$ occurs if and only if $\theta > \mu$, which is the "static no-growth" revolution condition familiar from Acemoglu and Robinson (2006). In the case that the high state is arbitrarily persistent ($q_H \to 1$), the maximal lump-sum tax is $\hat{T}_H = \frac{\delta}{1-\delta} \cdot Bh^r$ when $\phi = \phi_H^*$. In words, each poor agent (and rich agent) receives a transfer that is *as if* each elite gave Bh^r – his entire production less the productivity premium from being in the high state – to the group of poor agents, of which there are $1-\delta$. This is precisely the per-period income of each poor agent under revolution when costs vanish. Not surprisingly, both ϕ_L^* in the limit $q_H \to 1$ and ϕ_H^* in the limit $q_L \to 1$ are larger than this, which reflects, respectively, a premium for waiting for "good times" and a premium for falling permanently onto "bad times."

From a quantitative perspective, all of these limits are very large. In particular, if $B = 1$ so that there are no gains to production in the low state, Proposition 3 says that production in the high state must *more than double* one's wealth to avoid revolution when inequality is very high and revolt is nearly costless.

4.1.2 Case 2 (State-Dependent Constraints):

Now assume that the Markov process governing the economy's aggregate productivity is not effected by the revolution decision. It is easily verified that the poor agents' value functions for revolting in either state are

$$V^p(R(H)) = \frac{1-\mu}{1-\delta} \cdot \frac{BH}{1-\beta} \cdot \Big[1 + \frac{\phi(1-\beta q_L)}{1+\beta(1-q_L-q_H)}\Big] \tag{17}$$

and

$$V^p(R(L)) = \frac{1-\mu}{1-\delta} \cdot \frac{BH}{1-\beta} \cdot \Big[1 + \frac{\phi\beta(1-q_L)}{1+\beta(1-q_L-q_H)}\Big], \tag{18}$$

where it is clear that $V^P(R(H)) > V^P(R(L))$ from the second term in square brackets. It is then easy to verify that

$$R(s) \iff \begin{cases} \theta - \mu > (\mu - \delta) \cdot \frac{\phi(1-\beta q_L)}{1+\beta(1-q_L-q_H)}, & \text{if } s = H \\ \theta - \mu > (\mu - \delta) \cdot \frac{\phi\beta(1-q_L)}{1+\beta(1-q_L-q_H)}, & \text{if } s = L \end{cases}$$

so that the relative strength of either condition depends on whether $\delta > \mu$ or $\mu > \delta$. This is summarized in the following proposition.

Proposition 4: *When the payoffs to revolution are state-dependent,*

- ***High costs*** ***($\mu > \delta$):*** *Both constraints are strictly stronger than the no-growth revolution constraint $\theta > \mu$. Moreover, $R(H) \implies R(L)$ and the same comparative statics as in Proposition 2 obtain.*
- ***Low costs*** ***($\mu < \delta$):*** *Both constraints are strictly weaker than the no-growth revolution constraint $\theta > \mu$, but never hold for $\mu \geq \theta$. Moreover, $R(L) \implies R(H)$ and the comparative statics are opposite those in Proposition 2.*

Proof. See the Appendix. □

Whether $\frac{1-\mu}{1-\delta}$ is greater than one (low costs) or less than one (high costs) is crucial for poor agents' incentives to expropriate the capital stock. Intuitively, if this ratio is less than one, the per-capita gains to revolution are small in the sense that each poor agent receives less than one unit[32] of the aggregate capital – i.e., there is a high level of dispersion of the capital stock among the poor population. If the ratio is larger than one, the per capita gains are relatively larger.

[32] Of course, I really mean that the (Lebesgue) measure of each poor agents' share of the aggregate stock H is less than one, since capital is perfectly divisible and agents are atomistic.

4.1.3 The argument for Case 1

When should one expect either case to hold? First, in accordance with Acemoglu and Robinson's (2000) interpretation, let μ denote the real costs of coordinating, which is necessary for the poor agents to overcome the collective action problem and initiate a successful revolution.[33] If one accepts the Olsonian notion that larger groups are harder to coordinate (see Olson (1971)), then μ should increase with the size of the lower class. Formally, let $\tilde{\mu} : [0,1] \to [0,1]$ be a strictly decreasing and continuous bijection, so that the cost of coordinating $\tilde{\mu}(\delta)$ strictly increases with the size of the lower class. With these assumptions, there clearly exists a unique $\delta^* \in (0,1)$ such that

$$\tilde{\mu}(\delta) \begin{cases} > \delta, & \text{if } \delta < \delta^* \\ \leq \delta, & \text{if } \delta \geq \delta^*. \end{cases}$$

In words, for societies with a sufficiently large lower class, the coordination costs of revolution are sufficiently high to make comparative statics with state-dependent payoffs work in the same direction as those in Proposition 2.

On the other hand, if the cost μ does not depend on the relative class sizes – e.g., if μ is a measure of rule of law, with higher μ corresponding to better property rights and hence higher costs of expropriation – this argument fails, and the case $\delta > \mu$ may be relevant. The relative strength of the revolution conditions – namely, that revolution is more likely in the high state – is then comparable to "the voracity effect" in Tornell and Lane (1999), whereby positive shocks induce greater conflict over resources.[34] Intuitively, if costs to revolution are vanishingly small, it is always better to immediately grab the "bigger pie" when in the high state rather than accept the lump sum rebate $\hat{T}_H$ that is distributed among all agents, both rich and poor.

The assumption that the high state is never realized after revolution (Case 1) can be interpreted in a few ways. First, this formulation captures the notion that revolution is inherently harmful to productivity and growth. This could be for many reasons. For example, if the high productivity states are correlated with episodes of high levels of foreign investment or trade, it is reasonable to expect that civil conflict will cause foreign businesses to retreat. Similarly, law and order are likely to remain unenforced during periods of radical political transition, which would disincentive new investment and lower the pro-

[33]See Acemoglu and Robinson (2006) for a discussion of microfoundations.
[34]See also Benhabib and Rustichini (1996) and Benabou (1996).

ductivity of existing ones. All of these effects are consistent with the evidence on growth collapses outlined in Rodrik (1999) and and decreased investment during periods of political turbulence in Alesina and Perotti (1996). Second, this formulation can be understood as a statement about the managerial abilities of the poor agents. Even absent any of the distortions mentioned above, it is reasonable to think that existing capital and technology would be utilized less efficiently by poor agents. This justification is most compelling when the poor "masses" are relatively uneducated, e.g., at low levels of economic development.

Remark 2: *In the case of state-dependent constraints, limiting results analogous to those in Proposition 3 do not hold. Even restricting* $\mu \in [\delta, 1]$ *implies that* $\phi_s^* = \infty$ *for both states. This is because the value functions under revolution are now affine in* ϕ*, and hence scale without bound like the value functions under maximal redistribution. This is a useful observation, as it makes clear that the finite limits in Proposition 3 result from the opportunity cost (in terms of foregone lump-sum transfers) of revolution, which in that case is unbounded in* ϕ*.*

Remark 3: *I show in the Appendix that these state-dependent results are not robust to perturbations in the following sense. Assume that, after revolution, the economy falls into the low state for* $T-1$ *periods. In the* T^{th} *period, the Markov chain "picks up where it left off:" if revolution occurred in period* t^* *when* $s_{t^*} = H$*, then* $Pr(s_{t^*+T} = H) = q_H$ *(and analogously for the low state). Then, it is easy to show that if* $1 > \delta + \beta^{T-1}$*, all qualitative features are as in Propositions 2 and 3, and there is no problem sending* $\mu \to 0$*.*

As the comparative statics in Proposition 2 and the limit results in Proposition 3 seem realistic, and since reality is likely in between the state-independent and state-dependent cases as in Remark 3, I adopt the following assumption throughout the rest of the paper.

Assumption 1: *Payoffs to revolution are state-independent.*

4.2 An economy with accumulation and output growth

The only difference now is that I assume wealth does not depreciate between periods. Formally, per capita wealth (in the appropriate range – see below) follows the law of motion

$$h_{t+1} = \begin{cases} (1+\phi)h_t, & \text{if } s_{t+1} = H \\ h_t, & \text{if } s_{t+1} = L, \end{cases}$$

where for simplicity I normalize $B \equiv 1$ so that there is exactly zero growth in the low state.

It is easy to verify that the stationary distribution of the economy's transition matrix, defined by the fixed point equation

$$(m_H, m_L) \cdot \begin{pmatrix} q_H & 1-q_H \\ 1-q_L & q_L \end{pmatrix} = (m_H, m_L)$$

is given by

$$(m_H, m_L) = (\frac{1-q_L}{2-q_H-q_L}, \frac{1-q_H}{2-q_H-q_L}),$$

where m_s denotes the "average" fraction of time that the economy spends in state s.[35] Hence, the long-run average growth rate[36] $\hat{g}$ is given by

$$1 + \hat{g} = 1 + m_H \phi.$$

Following Acemoglu and Robinson (2000), which built on the seminal work of Galor and Zeira (1993), I assume a non-convexity in the accumulation technology.[37] Specifically, there exists $X > 0$ such that if $h_t < X$, then $h_{t+1} = h_t$ regardless of the aggregate state.

With only rich agents accumulating, the economy's aggregate growth rate is

$$\frac{H_{t+1}}{H_t} = \begin{cases} 1 + \phi\theta_t < 1 + \phi, & \text{if } s_{t+1} = H \\ 1, & \text{if } s_{t+1} = L \end{cases},$$

which is monotone increasing and unbounded. As before, $\theta_t = \frac{\delta h_t^r}{H_t}$ gives a measure of inequality at time t. Using this equation and observing that accumulation among the elites implies $H_{t+1}\theta_{t+1} = (1+\phi) \cdot H_t\theta_t$, it is easy to see that inequality follows the law of

[35] This "average" should be thought of as a sample average over infinitely many sample economies. That his notion coincides with the definition above follows from the (pointwise) ergodic theorem.

[36] This is assuming that all agents are accumulating – see below.

[37] Benabou (1996) develops an alternative method to model the aggregate effects of such credit constraints, but his specification only seems to be tractable under stringent distributional assumptions.

motion

$$\theta_{t+1} = \begin{cases} \frac{(1+\phi)\theta_t}{1+\phi\theta_t} > \theta_t, & \text{if } s_{t+1} = H \\ \theta_t, & \text{if } s_{t+1} = L, \end{cases}$$

where the first difference equation has fixed points $\theta^* \in \{0, 1\}$.

As shown in Section 4.1 for an endowment economy, large values of ϕ in the high state can prevent revolution even with a great deal of inequality. With growth, if poor agents are unable to accumulate, large ϕ results in rapidly growing inequality, bringing the revolution threat closer to the present. The objective of this section is to analyze how these two forces interact.

The first step is to characterize the relevant value functions. Let $V^i(s, h_t^i, \tau_t)$ denote the time t value function for an agent of type i who inherits wealth h_t^i from the previous period and who is taxed at rate τ_t after production takes place. Let $V^i(s, h_t^i, \tau \equiv 0)$ be the value function when $\tau_s = 0\ \forall s \geq t$. (The only difference from the case without accumulation is that now the value functions are time-varying.)

The following assumption is needed to ensure that things are well defined.

Assumption 3: $1 > \beta(1 + \phi)$.

This assumption says that the economy does not grow fast enough (relative to discounting) to make lifetime utility explode.[38]

The following lemma then gives part of the desired characterization.

Lemma 5: *Let Assumption 3 hold. When there is zero redistribution in all future periods, the value functions are given by*

$$V^i(s, h_t^i, \tau \equiv 0) = \frac{h_t^i}{1-\beta}, \text{ if } h_t^i < X,\ s \in \{L, H\},$$

$$V^i(H, h_t^i, \tau \equiv 0) = \frac{h_t^i \cdot (1+\phi) \cdot [1 - \beta(q_H - (1 - q_L))]}{1 - \beta q_L(1 - \beta(1+\phi)) - \beta(1-\beta)q_H(1+\phi) - \beta^2(1+\phi)}, \text{ if } h_t^i \geq X,$$

[38] Weaker conditions are possible, but this ensures well-defined value functions for all $(q_H, q_L) \in [0, 1] \times [0, 1]$. See the proof of Lemma 5 for details.

and

$$V^i(L, h_t^i, \tau \equiv 0) = \frac{h_t^i \cdot [1 - \beta(1+\phi)(q_H - (1-q_L))]}{1 - \beta q_L(1-\beta(1+\phi)) - \beta(1-\beta)q_H(1+\phi) - \beta^2(1+\phi)}, \text{ if } h_t^i \geq X.$$

Moreover,

- $V^i(H, h_t^i, \tau \equiv 0) > V^i(L, h_t^i, \tau \equiv 0)$
- $\frac{\partial V^i(H,h_t^i,\tau\equiv 0)}{\partial \phi} > \frac{\partial V^i(L,h_t^i,\tau\equiv 0)}{\partial \phi} > 0$
- $\frac{\partial V^i(H,h_t^i,\tau\equiv 0)}{\partial q_H} > \frac{\partial V^i(L,h_t^i,\tau\equiv 0)}{\partial q_H} > 0$
- $0 > \frac{\partial V^i(H,h_t^i,\tau\equiv 0)}{\partial q_L} > \frac{\partial V^i(L,h_t^i,\tau\equiv 0)}{\partial q_L}$
- $\frac{\partial V^i(H,h_t^i,\tau\equiv 0)}{\partial \beta} > \frac{\partial V^i(L,h_t^i,\tau\equiv 0)}{\partial \beta} > 0$

Proof. See the Appendix. □

Now, using Assumption 1 it is easy to see that the value function for poor agents staging a revolution in period t is

$$V^p(R(t)) = \frac{(1-\mu)}{1-\delta} \cdot \frac{H_t}{1-\beta},$$

where $H_t = \delta h_t^r + (1-\delta)h_t^p$. If $h_0^p < X \leq h_0^r$ and there are no transfers, clearly positive growth implies the existence of $t^* \in \mathbb{N}$ such that $V^p(R(t)) > V^p(s, h_t^p, \tau \equiv 0)$ if and only if $t \geq t^*$ and where $h_s^p = h_0^p \; \forall s < t^*$. That is, aggregate growth and different rates of accumulation between socioeconomic groups implies that the revolution constraint will bind in finite time (except for a corner case). The following lemma documents this observation. The expectation operator $\mathbb{E}_{\vec{m}}$ is taken with respect to the measure induced by the ergodic distribution (m_H, m_L).

Lemma 6: *Let $h_0^p < X$ and $1 > \mu \geq \theta_0 > 0$. If $q_L < 1$, then $\mathbb{E}_{\vec{m}}(t^*) < \infty$ and is decreasing in ϕ, q_H, and $1 - q_L$. If $q_L = 1$, $\mathbb{E}_{\vec{m}}(t^*) = \infty$.*

Proof. See the Appendix. □

The (pointwise) Ergodic Theorem tells us that the same results obtain if we were to take sample averages over (arbitrarily large subsets of) a sequence of identical economies $\{e_i\}_{i \in \mathbb{N}}$. I emphasize that the cutoff time t^* should *not* be interpreted as calendar time.

Rather, it should be interpreted as the *length of time since the economy began to experience "modern" (i.e., industrial or post-industrial) growth.*

The next thing that needs to be determined is whether a one-period transfer can push poor agents' wealth over the threshold X so that they can accumulate in future periods. If yes, inequality will decrease, and with no subsequent transfers, will stay at this level for all time. Inequality will of course decrease again if any subsequent transfers are made.

If $s_t = L$, only null transfers can be made, so poor agents remain below the accumulation threshold. If $s_t = H$, poor agents remain unable to accumulate if and only if

$$(1 - \hat{\tau}) \cdot h_t^p + \hat{\tau} \cdot H_t < X,$$

which, recalling the law of motion for aggregate wealth, that $h_t^p = h_0^p$, and the definition of $\hat{\tau}$ simplifies to

$$\frac{h_0^p}{1+\phi} + \frac{\phi}{1+\phi} \cdot H_{t-1} \cdot [1 + \phi\theta_{t-1}] < X.$$

The following fact is immediate from the observation that the second term on the left is unbounded in ϕ and H_{t-1}.[39]

Lemma 7: *Suppose $h_0^p < X$, $s_t = H$, and no transfers have been made prior to date t. Then,*

- *For all $X, \tau, H_{t-1} > 0$ and $\theta_{t-1} \in [0,1]$, there exists $\hat{\phi} > 0$ such that $\phi \geq \hat{\phi}$ implies that poor agents can begin to accumulate in period t.*
- *For all $X, \tau, \phi > 0$ and $\theta_{t-1} \in [0,1]$, there exists $\hat{H} > 0$ such that $H_{t-1} \geq \hat{H}$ implies that poor agents can begin to accumulate in period t.*

In words, in the good state (so that positive transfers are feasible), poor agents with arbitrarily few assets can begin to accumulate if either growth or aggregate wealth is large enough. In this sense, wealth levels and the rate of wealth growth are substitutes.

The following proposition gives a partial characterization of when a society can avoid revolution, given that poor agents have not yet been able to accumulate. The focus is on one-time transfers and limiting cases, since these are the easiest to deal with and have clear

[39]That the poor are taxed despite getting zero net returns on production is not totally consistent with the reasoning that led to the maximal tax rates $\hat{\tau}_{st}$, but the calculations are somewhat cleaner in this case. All the following results continue to hold if the poor are not taxed when they cannot accumulate.

intuitive meaning. By the Markovian assumptions on strategies and the aggregate state transition matrix, it is sufficient to suppose $t = 0$.

Proposition 5: *Let Assumption 3 hold, let $h_0^p < X < h_0^r < H_0$ be given, and suppose that $V^p(R(0)) > V^p(s, h_0^p, \tau \equiv 0)$. Then,*

1. *If $s_0 = L$, then for all $\phi > 0$ there exists $q_L^* \in (0,1)$ such that $q_L > q_L^* \implies R(0)$. That is, no transfer scheme can avoid immediate revolution.*

2. *If $s_0 = H$, there exists $\phi^* > 0$ such that $\phi \in [0, \phi^*) \implies R(0)$. That is, no transfer scheme can avoid immediate revolution.*

3. *If $s_0 = H$ and $\min[X, \frac{1-\mu}{1-\delta} \cdot H_0] > \frac{h_0^p}{1+\phi} + \frac{\phi}{1+\phi}(1 + \phi\theta_0)H_0$, then there exists $(q_H^*, q_L^*) \in (0,1) \times (0,1)$ such that revolution occurs immediately if $q^H < q_H^*$ and $q_L > q_L^*$.*

4. *If $s_0 = H$ and $X > \frac{h_0^p}{1+\phi} + \frac{\phi}{1+\phi}(1 + \phi\theta_0)H_0 \geq \frac{1-\mu}{1-\delta} \cdot H_0$, revolution in the current period can be avoided but poor agents remain unable to accumulate. This is possible only if $\mu > \delta$ and β is sufficiently small. Revolution is then avoided in the subsequent period without additional transfers if and only if $\mu \geq \theta_1 = \frac{\theta_0}{1+\phi\theta_0} + \frac{\delta\phi}{1+\phi}$, which can hold only if θ_0 and μ are sufficiently large.*

5. *If $s_0 = H$ and agents are sufficiently impatient ($\beta \approx 0$), then for all $q_H \in [0,1]$ there exists $\phi^* > 0$ such that $\phi \geq \phi^* \implies$ revolution* never *occurs when transfers are only for one period.*

Proof. See the Appendix. □

The first three cases yield pessimistic conclusions about social conflict in unequal societies with stagnant economies. Case (1) should be interpreted as follows. Consider an economy with a potentially very high growth rate that has succumbed to a period of stagnation. This could be due to, e.g., sectoral stagnation, an externally-induced growth collapse, or an sharp business cycle downturn that induces enough uncertainty to make agents unsure about whether it will ever be escaped.[40] This part of the proposition says that, if agents believe that bad times will be very persistent, no amount of potential growth can relieve immediate social tensions. Case (2) is similar, but applies to economies in which the

[40] An example of an externally-induced growth collapse is the breakdown of relations with a major trading partner. Regarding the final possibility, I explicitly introduce uncertainty in a later section.

maximum potential growth is very low. Finally, Case (3) says that moderate but transient windfalls are not sufficient to mitigate the revolution threat.

The latter two cases represent the main theme of this paper – that rapid growth can relieve social tensions. In Case (4), growth is high enough to relieve the immediate revolution threat, but not high enough to allow poor agents to begin accumulation. If initial inequality is high enough, this may be enough to prevent revolution in the subsequent period even without additional transfers. Finally, Case (5) says that, even if growth is transitory, if it large enough it can prevent the revolution threat for the rest of time. This happens because maximal transfers relax the immediate revolution threat while simultaneously pushing poor agents over the accumulation threshold. Therefore, absent future transfers, inequality will remain constant in future periods because both classes of agents accumulate at the same rate.[41]

5 Commitment and democratization

I have thus far abstracted from commitment problems. The elite were assumed, at least implicitly, to be able to commit to maximal transfers in every subsequent period; whether revolution could be avoided then depended only on economic fundamentals. There are two ways to introduce commitment problems for the elite: (1) intra-period hold-up and (2) the inability to commit intertemporally. Since it turns out that intra-period hold-up (i.e., when the tax rate is set after the revolution decision) is not very interesting because zero redistribution is always a dominant strategy for elites, I follow Acemoglu and Robinson (2000) and focus on the intertemporal commitment problem.

5.1 Democracy in the baseline model

In the previous sections, there has been no difference between democratization and full redistribution ($\tau_t = \hat{\tau}_{s_t}\ \forall t$) under elite control. Under democracy, since the median agent

[41] Observe that, for Cases (4) and (5) to hold, β must be sufficiently small. This is a technical result that follows from Assumption 3, and is necessary to ensure that lifetime utilities remain finite. This fact also makes it difficult to find a tractable sufficient condition for growth to mitigate the revolution threat in the limit of perfect patience, $\beta \to 1$. Indeed, Assumption 4 implies that $\beta \to 1$ implies that $\phi \to 0$, which brings us back to Case (2) and immediate revolution. Perhaps the most appealing interpretation of this result is that $\beta \to 1$ represents the limit of short period lengths and frequent actions. Under this interpretation, the fact that the revolution threat binds today means that it will also bind in an arbitrarily large number of subsequent periods, and hence it may as well be acted on today. While this result is plausible, I suggest that it is best thought of as a technical artifact.

is poor and the policy space (the set of feasible tax rates) is one-dimensional, the Median Voter Theorem implies that $\tau_t = \hat{\tau}_{s_t}\ \forall t$. Since I have implicitly assumed that the elites have the power to commit to such taxation paths, redistribution under democracy can be no better for poor agents. I can therefore restate parts 1 and 2 of Proposition 5 as follows.

Proposition 5': *When there is a high degree of economic inequality and the poor are credit-constrained, peaceful democracy cannot be supported when the low state is highly persistent or when growth rates are very low.*

In this context, "revolution" cannot be interpreted in the political sense, for poor agents already control the political system. Instead, it should be thought of as costly social or political conflict – either by direct expropriation of wealth (through civil conflict, riots, etc.) or by large-scale political and economic overhauls. This result therefore says that *even democratic societies are prone to suffer from social conflict when inequality is high and growth is low or transient.* This is consistent with the analysis in Friedman (2006, 2009).

On the other hand, the following result yields the positive conclusion that, if economic fundamentals are sufficiently good to support democracy, the wealth distribution will converge to perfect equality in the long run. As the economy continues on its development path, poor agents are able to start accumulating. After this point, redistributive policies let them "catch up" with the elites on a per capita basis.

Proposition 6: *If revolution can be avoided and maximal redistribution continues indefinitely,* $\lim_{t\to\infty} \theta_t = \delta$. *That is, the economy converges to perfect equality. Moreover, this convergence is monotone.*

Proof. See the Appendix. □

The more interesting cases are when the feasible redistribution schedules under democracy and elite control are distinct due to commitment issues. This is the issue I take up in the next subsection.

5.2 Intertemporal commitment problems

Suppose now that, as in Acemolgu and Robinson (2000, 2006), the revolution threat is stochastic in the following sense. In addition to the aggregate growth state, in each period the poor are either coordinated (c) or not coordinated (n). Denote this state variable by $r_t \in \{c, n\}$, which is distributed i.i.d. in each period with $\Pr(r = c) = p$. The costs to revolution now depend on the variable r_t, i.e., with $\mu(c) = \mu$ and $\mu(n) = 1$. Hence, when poor agents are coordinated, payoffs are as described previously and the revolution threat is real. When poor agents are not coordinated, all output is destroyed during revolution, meaning that revolt is always strictly dominated. This specification implies that the elites have a commitment problem: when $r_t = n$, it is always optimal for them to set $\tau = 0$.[42]

Let $(s_t, r_t) = (H, c)$. Clearly if $p = 0$, the value function for poor agents in this case is equal to

$$(1 - \tau_t)h_t^p + T_t + \beta q_H V^p(H, h_t^p + T_t, \tau \equiv 0) + \beta(1 - q_H)V^p(L, h_t^p + T_t, \tau \equiv 0)$$

since no transfers will be received in the future. If $p = 1$, there is no commitment problem and the value function is the same as in the previous section under maximal redistribution (see, e.g., the proof of Proposition 5).

The first observation is that, when $s_t = L$, current transfers are equal to zero regardless of the value of r_t; this follows from my assumptions on the distribution technology. The commitment problem therefore reduces poor agents' expected payoffs in the low state only through expected transfers when the high state is realized. This suggests the following fact, the proof of which is omitted because it is trivial.

Lemma 8: *Under the same conditions as Proposition 5 and if $s_0 = L$, there exists $q_L^{**}(p) \in (0, 1)$ such that $q_L > q_L^{**}(p)$ implies immediate revolution. This cutoff level is strictly increasing in p with $q_L^{**}(0) = 0$ and $q_L^{**}(1) = q_L^*$, where q_L^* is defined in part 1 of Proposition 5.*

In words, more severe commitment problems can induce revolution in the low state, even when it is not very persistent. Complete lack of commitment induces immediate revolution even when the low state is completely transient ($q_L = 0$), since no redistribution will take place in the high state. An analogue of part 3 of Proposition 5 can be similarly obtained.

More generally, I want to answer the question: *In this environment, when will the elites choose to democratize to mitigate the revolution threat?* Clearly, the fundamentals

[42] Clearly there are other ways to model such commitment issues. The stochastic revolution threat is analytically convenient and allows for direct comparison with extant results.

(ϕ, q_H, q_L) must be "strong enough" to prevent revolution when there is maximal redistribution in every period, since this is what will occur when poor agents select the tax rate under democracy. In addition, as in Acemoglu and Robinson (2000) the probability that a revolutionary threat materializes p must be low enough that revolution would occur without democratization because the promise to continue redistribution in the future is not credible. The remaining question is then how the critical value p^* that makes poor agents indifferent between revolution and accepting elite control depends on the fundamentals (ϕ, q_H, q_L)? The intuition behind Lemma 8 suggests the following fact.

Proposition 7: *Suppose that poor agents prefer permanent, maximal redistribution to revolution and that one-period transfers cannot prevent revolution. The cutoff value p^* is*

- *Decreasing in ϕ and q_H.*
- *Increasing in q_L.*

That is, for $p < p^*$ the revolution threat is met by democratization. If $p \geq p^*$, elites prevent revolution through (temporary) redistribution.

Proof. See the Appendix. □

This result mirrors the findings in Acemoglu and Robinson (2000), and suggests that peaceful regime changes from an authoritarian regime to democracy is more likely in (moderately) bad economic times. If the economic fundamentals are bad enough, revolution (or, in an alternate interpretation, large-scale wealth redistribution) will dominate democratization for the poor agents. It is possible to solve for explicit cutoff values in the case of no accumulation (the case with accumulation is more difficult), but since the comparative statics are obvious I omit the explicit characterization.

The main results obtained up to this point are summarized below for convenience.

Main Predictions:

- *If the elites hold political power and suffer from an intertemporal commitment problem, the threat of revolution is met with democratization if (1) the economic fundamentals are not too bad, and (2) the commitment problem is severe.*

- *Under the same conditions, the revolution threat is met with voluntary transfers under elite control if the commitment problem is not too severe. Good economic fundamentals decrease the relative severity of the commitment problem.*

- *Under the same conditions, or if democracy has already been established, revolution occurs when inequality is high and the economic fundamentals are bad.*

- *Rapid and persistent growth implies that the observable actions enumerated above will occur on-path at an earlier (expected) date.*

6 Conclusion and suggestions for future work

This paper has developed a game-theoretic model of inequality, redistribution, and political conflict in an economy undergoing sustained, stochastic growth. The main insights are generally consistent with stylized empirical and historical facts about political economy and growth. Namely, more rapid growth makes tensions related to inequality and wealth distribution salient earlier along the economy's development path. At the same time, the expectation of robust and rapid future growth also provides the means for resolving such tensions without on-path conflict. Finally, the viability of non-democratic political institutions is a function not only of the leaders' ability to commit, but also of the current and (expected) future economic climate.

There are number of ways to extend the model developed here. Modeling agents as Ramsey consumers is one, for example. More interesting, I believe, would be to endogenize the growth mechanism, which in this paper was treated as exogenous. Acemoglu (2009, 2010) partially accomplishes this goal, but is still rooted in neoclassical growth theory and does not allow for long-run growth. I also think it is important to incorporate social mobility into the present framework. Drawing on the insights of Benabou and Ok (2001), one can see how expectations about the prospects for upward mobility (which are correlated with the economy's aggregate growth rate) would actually decrease the demand for redistribution, further amplifying the positive effects of growth highlighted in this paper. On the other hand, as Acemoglu and Robinson (2002) allude to, downward shocks to expectations about growth would then also correlate with downward shocks to expectations about mobility prospects. Such a mechanism would provide a powerful – and I believe realistic – explanation for the establishment of social insurance in the U.S. during the Great Depression – over a century after the initial establishment of a constitutional democracy.[43] Modeling mobility is challenging, however, for it requires the modeler to keep track of the entire (ergodic) income distribution.

Another obvious avenue for further research is a more comprehensive empirical investigation of the model's validity. Numerous historical studies examine, for example, the dynamics of the income distribution and aggregate growth in nineteenth century Britain during the Industrial Revolution.[44] But, by and large, the results are inconclusive or contradictory. To the best of my knowledge, no comprehensive empirical study has examined these economic variables in conjunction with political reforms across a wide range of developing countries, though this is likely due to data availability (and reliability) issues. To study the model's contemporary relevance, a study using opinion data (such as the World Values Survey) and consumer expectation survey data to study the link between growth expectations and political satisfaction could be particularly interesting.

Moreover, the representations of "democracy" and "conflict" that I have chosen in this paper are admittedly narrow. In particular, oftentimes social conflict is not a "rich vs. poor" phenomenon; indeed, economic inequality may not be the (primary) cause. In general, it is not clear that a country's "level of democracy" is even the best measure for sociopolitical openness and cohesion. Historically speaking, even democracies are often home to censorship, discrimination, and disenfranchised subpopulations. Friedman (2006, 2009), for example, emphasizes that economic stagnation often instigates anti-immigrant sentiments due to concerns about the labor market, religious intolerance, and ethnic clashes. Sheve and Slaughter (2001) provides a more systematic empirical analysis of the first issue in a contemporary context. For the latter two, the evidence in Rodrik (1999) indicates that non-economic – and, in particular, ethnic – divisions are a major correlate with a country's sensitivity to negative growth shocks. I believe that these are likely fruitful areas for future theoretical work.

In ongoing work,[45] I develop a complementary political economy model based on a simple labor search framework, as in Austen-Smith and Wallerstein (2003). When economic growth is strong, the labor market is ripe with "good" jobs, which grant workers economic rents. But when times are bad, these positions are scarce. When there is heterogeneity among the workforce that is observable and that results in asymmetric political power –

[43] Both Acemoglu and Robinson (2002) and Friedman (2006) note that the U.S.'s experience during the Great Depression is somewhat anomalous with respect to their theories.

[44] Refer to Acemoglu and Robinson (2000) and Friedman (2006) for a wide range of references.

[45] This was omitted for the sake of thematic continuity and space constraints.

due to, e.g., ethnic differences, or the difference between native citizens and immigrants – it is optimal for the politically enfranchised groups to block those without political sway.[46] This drives output down even further, and can result in a "growth trap" with low economic activity and large political economy distortions, providing microfoundations for the reduced-form framework in Caplan (2003). One can also investigate incentives for other interest groups to form blocking coalitions to break out of this vicious circle.[47] These incentives depend on the mechanisms for surplus division between matched workers and firms in the labor market.

As discussed in the introduction, incorporating insights from psychology and behavioral economics into the present political economy setting would likely result in novel insights. The self-signaling framework developed in Benabou (2008), for example, seems like an appropriate starting point for such work.

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[46] Acemoglu (2009) obtains a related set of results in a model in which oligarchs block entrepreneurial entry and thereby partially shut off the creative destruction mechanism that drives high levels of output.

[47] This is similar in spirit to Benabou et al (2013), which studies a very different economic problem.

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8 Appendix

8.1 Derivations

Derivation of the value functions in Section 3: Observe from the recursive equations that Equation (11) is immediate. It remains to establish Equation (10). Plugging Equation (11) into Equation (8) and moving all $\hat{V}_H^P$ terms to the left, we get

$$\hat{V}_H^P\Big[1-\beta q_H-\beta(1-q_H)\cdot\frac{\beta(1-q_L)}{1-\beta q_L}\Big]=Bh^P+\phi BH+\beta(1-q_H)\frac{BH(\frac{1-\theta}{1-\delta})}{1-\beta q_L},$$

where I've used the definitions of A_H and $\hat{\tau}_H$. The term in brackets on the left simplifies to $\frac{(1-\beta)[1+\beta(1-q_L-q_H)]}{1-\beta q_L}$. Substituting $h^P=\frac{1-\theta}{1-\delta}H$ and collecting terms, the RHS becomes BH times

$$\frac{1-\theta}{1-\delta}\cdot\frac{1+\beta(1-q_L-q_H)}{1-\beta q_L}+\phi.$$

Dividing through by the factor on the LHS yields the desired result.

Derivation of the post-revolution value functions in Section 3.2: The value functions in this case must satisfy the recursive equations

$$V^P(R|H)=\frac{(1-\mu)A_H H}{1-\delta}+\beta\Big[q_H V^P(R|H)+(1-q_H)V^P(R|L)\Big]$$

and

$$V^P(R|L)=\frac{(1-\mu)BH}{1-\delta}+\beta\Big[q_L V^P(R|L)+(1-q_L)V^P(R|H)\Big].$$

The remaining calculations are analogous to those in the previous derivation.

8.2 Proofs:

Proof of Lemma 1: Poor agents in period t revolt if and only if $y_t^p(R(t))>y_{t+1}^p$. In equilibrium, there must be no revolt when they are indifferent or else there is an open set problem for the elites. Because they receive nothing after a revolution, elites optimally choose the largest surplus share consistent with no revolt. If feasible, the optimal level is $\frac{(1+\phi)\mu-\theta_t}{\phi} \in [0,1]$. If this quantity is negative, redistribution sufficient to mitigate the revolution threat is infeasible: $y_t^p(R(t)) > y_t^p$ and revolt occurs. The only indeterminacy results from the fact that, when revolution is unavoidable, any offer $\alpha_t \in [0,1]$ can be offered. Since all of these strategies are outcome-equivalent, I refer to the equilibrium as essentially unique. *Q.E.D.*

Proof of Proposition 1: Follows immediately from the proof of Lemma 1 and the law of motion for inequality. In particular, because of the linear production technology and linear preferences, inequality jumps to its steady state level after a single period. *Q.E.D*

Proof of Proposition 2: (1), (3), (4) follow from direct calculation. (2) follows from $\beta(1-q_L) < (1-\beta q_L)$. *Q.E.D.*

Proof of Corollary 1: Let $f(L)$ denote the expression on the RHS of the $R(L)$ constraint. Define $f(H)$ analogously. Then direct calculation yields $\frac{\partial f(H)}{\partial q} = -\frac{\partial f(L)}{\partial q} > 0$. *Q.E.D.*

Proof of Proposition 3: Item (1) in Proposition 2 reveals that $\phi_H^* = \frac{1+\beta(1-q_L-q_H)}{(1-\beta q_L)(1-\delta)}$ and $\phi_L^* = \frac{1+\beta(1-q_L-q_H)}{\beta(1-q_L)(1-\delta)}$. The result follows from taking the appropriate limits of these expressions. *Q.E.D.*

Proof of Proposition 4: The right hand side of either constraint is positive if and only if $\mu > \delta$. The comparative statics follow immediately from this observation and comparison with Proposition 2. Clearly there is never revolution in the high cost case when $\mu \geq \theta$ since the right hand sides of both constraints are strictly positive. In the low cost case, the right hand side is strictly negative, but if either constraint held we would have $\mu \geq \theta > \delta > \mu$, a contradiction. *Q.E.D.*

Proof of Lemma 5: The case where $h_t^i < X$ is trivial, as it is just the perpetuity value of this quantity. When $h_t^i \geq X$, the value functions must satisfy the recursive equations

$$V^i(H, h_t^i, \tau \equiv 0) = (1+\phi)h_t^i + \beta\Big[q_H \cdot V^i(H, (1+\phi)h_t^i, \tau \equiv 0) + (1-q_H) \cdot V^i(L, (1+\phi)h_t^i, \tau \equiv 0)\Big] \tag{19}$$

and

$$V^i(L, h_t^i, \tau \equiv 0) = h_t^i + \beta\Big[q_L \cdot V^i(L, h_t^i, \tau \equiv 0) + (1-q_L) \cdot V^i(H, h_t^i, \tau \equiv 0)\Big]. \tag{20}$$

The assumption $B \equiv 1$ is convenient here, as it allows us to solve the second equation in terms of $V^i(H, h_t^i, \tau \equiv 0)$ as follows:

$$V^i(L, h_t^i, \tau \equiv 0) = \frac{h_t^i + \beta(1-q_L) \cdot V^i(H, h_t^i, \tau \equiv 0)}{1-\beta q_L}. \tag{21}$$

In order to solve the first recursive equation, I guess that $V^i(s, \cdot, \tau \equiv 0)$ is homogenous of degree 1 and verify that it gives an appropriate solution. Using this guess and substituting (18) into (16), we get

$$V^i(H, h_t^i, \tau \equiv 0) = (1+\phi)h_t^i + \beta\Big[q_H \cdot (1+\phi) \cdot V^i(H, h_t^i, \tau \equiv 0) + (1-q_H) \cdot (1+\phi) \cdot \{\frac{(1+\phi)h_t^i + \beta(1-q_L)\cdot(1+\phi)\cdot V^i(H, h_t^i, \tau \equiv 0)}{1-\beta q_L}\}\Big]. \tag{22}$$

The desired representation of $V^i(H, h_t^i, \tau \equiv 0)$ then follows from tedious algebra, and the representation of $V^i(L, h_t^i, \tau \equiv 0)$ follows from substituting that solution into (18). It is then easy to verify that these solve the required recursive equations. To ensure that these are the unique solutions (and that the homogeneity assumption is legitimate), note that all the conditions of Assumption 1 in Alvarez and Stokey (1998) are satisfied, with the per-period utility function $u(c) = c$ homogenous of degree 1.

Now I show that Assumption 3 holds if and only if both numerators and the denominators are positive $\forall (q_L, q_H) \in [0,1] \times [0,1]$, which is necessary to ensure that we haven't divided through by zero. The numerator part is clear; the $V^i(H, h_t^i, \tau \equiv 0)$ is the tighter constraint. Observe that the denominators are decreasing in ϕ, and the unique number ϕ^* that sets the denominators equal to zero is

$$\phi^* = \frac{(1-\beta)(1-\beta(q_H - (1-q_L))}{\beta[(1-\beta)q_H + \beta(1-q_L)])},$$

which is decreasing in q_H and increasing in q_L. Hence, evaluating this expression at $(q_L, q_H) = (0,1)$ yields that $\phi < \frac{1-\beta}{\beta}$ if and only if the denominator is positive $\forall (q_L, q_H) \in [0,1] \times [0,1]$. This is exactly the condition in Assumption 3.

The comparative statics follow from differentiation. The computations are tedious but straightforward, so they are omitted. The only nontrivial steps involve repeatedly using Assumption 3 to sign expressions. *Q.E.D.*

Proof of Lemma 6: When poor agents are unable to accumulate, the revolution constraint reduces to $\theta_t > \mu$. It remains to find the expected law of motion for θ under the measure induced by $\vec{m}$. Under this measure, we have $\theta_{t+1} = m_H \cdot \frac{1+\phi}{1+\phi\theta_t} \cdot \theta_t + (1-m_H) \cdot \theta_t$.

Suppose $q_L < 1 \iff m_H > 0$. Then the law of motion has fixed points $\theta^* \in \{0,1\}$. Since it is monotone increasing in θ_t and $\theta_0 > 0$ by assumption, it must tend toward the upper fixed point. It is easy to check that the rate of convergence is increasing in ϕ and m_H. Since m_H is increasing in q_H and decreasing in q_L, it follows that $\frac{\theta_{t+1}}{\theta_t}$ is as well. The hitting time t^* is trivially decreasing in $\frac{\theta_{t+1}}{\theta_t}$, yielding the appropriate comparative statics. Finiteness follows from the fact that $\mu < 1$ and that θ_t will hit any open neighborhood of 1 in finite time.

Suppose $q_L = 1$. Then the only fixed point to the law of motion is $\theta^* = \theta_0 \leq \mu$. *Q.E.D.*

Proof of Proposition 5:
Part 1: Let $s_0 = L$. Since only null transfers are possible, poor agents can begin to accumulate no sooner than $t = 1$. Let $\hat{V}^p(s_t, h_t^p)$ denote the value function for poor agents in state s_t with wealth $h_t^p < X$ when $\tau_t = \hat{\tau}\ \forall s \geq t$. For $s_t = L$ this must satisfy the recursive equation

$$\hat{V}^p(L, h_t^p) = h_t^p + \beta q_L \hat{V}^p(L, h_t^p) + \beta(1 - q_L)\hat{V}^p(H, h_t^p).$$

By simple algebra and continuity we have

$$\lim_{q_L \to 1} \hat{V}^p(L, h_t^p) = \frac{h_t^p}{1 - \beta},$$

which is precisely equal to $V^p(L, h_t^p, \tau \equiv 0)$ for $h_t^p < X$. Since by assumption $V^p(R(0)) > V^p(L, h_0^p, \tau \equiv 0)$, there must exists $q_L^* \in (0, 1)$ such that $V^p(R(0)) > \hat{V}^p(L, h_0^p)\ \forall q_L > q_L^*$. But this is the definition of $R(L, 0)$, so this part of the proposition is proved.

Part 2: Let $s_0 = H$. When $\phi = 0$, the value function for poor agents under maximal redistribution is identical to $V^p(H, h_0^p, \tau \equiv 0)$ since positive redistribution is never feasible. Since the revolution constraint binds by hypothesis, there is immediate revolution. If the value function under maximal redistribution is continuous in ϕ (from the right at $\phi = 0$), the claim is proved.

That this is true follows from the Lebesgue Dominated Convergence Theorem (LDCT). Write lifetime utility under maximal redistribution in the series form

$$\mathbb{E}[\sum_{t=0}^{\infty} \beta^t((1 - \hat{\tau}_{s_t})h_t^p + \hat{T}_{s_t})|s_0 = H].$$

This should be viewed as a double integral over $\{L, H\}^{\mathbb{N}} \times \mathbb{N}$ (endowed with the discrete sigma algebras) with respect to the product measure over sequences of states and time. Formally, this measure is $\mathbb{P} := \mathbb{Q} \times \mu$, where $\mathbb{Q}$ is induced on the appropriate sequence space by the Markov transition kernel and μ is the counting measure on $\mathbb{N}$. Though I've written it as an iterated integral, it is of course the same as the double integral described above by Tonelli's Theorem (the measure spaces are countably generated and clearly σ-finite).

Now, consider any sequence of functions $h^k : \{L, H\}^{\mathbb{N}} \times \mathbb{N} \to \mathbb{R}_+$ of the form $h^k(H, t) := \frac{1}{1+\phi^k} h_{t-1}^k + \frac{\phi^k}{1+\phi^k}(1 + \phi^k \theta_{t-1})H_{t-1}$ and $h^k(L, t) := h^k(\cdot, t - 1)$, where $\phi^k \to 0$. Since under any redistributive scheme $\theta_t > \delta\ \forall t < \infty$ (see the proof of Proposition 6), clearly $h^k(s_t, t) < H_t\ \forall t \in \mathbb{N}$. Under Assumption 3, this dominating function is integrable in the above sense. (This follows from the equivalence of the sequence and recursive formulations and the cited results in Alvarez and Stokey (1998).) Continuity then follows from the

LDCT.[48]

Part 3: When $s_t = L$, redistribution in period t is zero. Hence, at $q_L = 1$ and $q_H = 0$, the value function in $s_0 = H$ under maximal redistribution is equal exactly to the perpetuity value of h_1^p, since poor agents are assumed to not be able to accumulate after this one-time transfers. This returns the revolution constraint, which holds by assumption. The result follows from continuity of the value functions in the transition probabilities. (This can be seen from the recursive equations that they must satisfy.)

Part 4: Let $s_0 = H$. Revolution can be avoided without accumulation if

$$X > \frac{h}{1+\phi} + \frac{\phi}{1+\phi}(1+\phi\theta)H \geq \frac{1-\mu}{1-\delta} \cdot H.$$

The middle expression is increasing, unbounded, and continuous in ϕ, so there clearly exists an open interval $\mathcal{I} \subset \mathbb{R}_{++}$ such that the above inequalities are satisfied for $\phi \in \mathcal{I}$. The necessary conditions are obvious. To obtain the expression for θ_1, note that under maximal transfers

$$\theta_1 \cdot H_0 \cdot (1+\phi\theta_0) = \theta_1 \cdot H_1 = \delta h_1^r = \delta \cdot [h_0^r + \frac{\phi}{1+\phi}(1+\phi\theta_0)H_0].$$

Dividing through by $H_0 \cdot (1+\phi\theta_0)$, we get

$$\theta_1 = \delta \cdot [\frac{\theta_0}{\delta(1+\phi\theta_0)} + \frac{\phi}{1+\phi}] = \frac{\theta_0}{1+\phi\theta_0} + \frac{\delta\phi}{1+\phi}.$$

Because the poor cannot accumulate in period 1 by assumption, the revolution constraint when there are no further transfers reduces to $\theta_1 > \mu$, i.e., $\mu \geq \theta_1$ is necessary and sufficient for revolution to be avoided without additional transfers in $t = 1$. A necessary condition for this is $\theta_0 > \theta_1$. Note that $\theta_0 > \theta_1$ if and only if $\frac{\theta_0^2}{1+\phi\theta_0} > \frac{\delta}{1+\phi}$; a sufficient condition is $\theta > \frac{1}{\sqrt{2}}$ since $\delta < \frac{1}{2}$.

Part 5: This is similar to the proof or Part 4. Denote $\tilde{h}^p(\phi) := \frac{h}{1+\phi} + \frac{\phi}{1+\phi}(1+\phi\theta)H$ so that revolution is avoided and accumulation occurs if and only if $\tilde{h}^p(\phi) \geq \max\{X, \frac{1-\mu}{1-\delta} \cdot H_0\}$. When $\beta = 0$, $\tilde{h}^p = V^p(H, \frac{\tilde{h}^p}{1+\phi}, \tau \equiv 0)$. Moreover, $\beta = 0$ makes the restriction of Assumption 3 vacuous, so ϕ is not bounded above and this inequality can be satisfied. Continuity (from the right) of $V^p(s, \cdot, \tau \equiv 0)$ in β at $\beta = 0$ yields that the same statements hold for small β. Since both types begin to accumulate, in the absence of future transfers inequality is constant; hence the revolution constraint never binds again.

[48] It is easy to show (right) continuity of the value function at $\phi = 0$ in the recursive formulation when (q_H, q_L) is in a neighborhood of $(0, 1)$. I have not found an easy proof that extends this result to all $(q_H, q_L) \in [0, 1] \times [0, 1]$.

To see why β must be small, consider the following argument. For fixed ϕ, this value function is increasing in β. But since ϕ is bounded above by a function of β by Assumption 3, we must substitute $\phi = \frac{1-\beta}{\beta} - \epsilon$ for ϵ small to get the maximum acceptable growth rate. Then, we must differentiate with respect to β. At exactly $\phi = \frac{1-\beta}{\beta}$ (which is not a problem for (q_H, q_L) on the interior of the unit square), the value function $V^p(s, h, \tau \equiv 0) = \frac{h \cdot [1-\beta(q_H-(1-q_L))]}{\beta(1-\beta)(1-q_H)}$. This decreases until $\beta = \frac{1}{2}$ and increases thereafter. But, for $\beta \approx 1$, when the value function becomes large again, the maximal ϕ becomes small by Assumption 3 and it is possible that poor agents will not be transferred enough to begin accumulation in the first place. Obtaining a nontrivial sufficient condition in that case is somewhat unwieldy, so I do not give one. *Q.E.D.*

Proof of Proposition 6: When $s_t = L$, there is no accumulation and no redistribution, so it suffices to only consider the case $s_t = H$. (The fraction of time spent in the low state only slows down the speed of convergence.) Since elites are assumed to be able to accumulate in the initial period and the aggregate economy is growing at a positive rate, it is easy to see that poor agents must begin accumulation in finite time under maximal redistribution. Under maximal transfers and when both types accumulate, in the high state we have

$$\theta_t \cdot H_{t-1} \cdot (1+\phi) = \theta_t \cdot H_t = \delta h_t^r = \delta \cdot [h_{t-1}^r + \frac{\phi}{1+\phi}(1_\phi) H_t].$$

Rearranging, we see that in the high state θ follows the law of motion

$$\theta_{t+1} = \frac{\theta_t}{1+\phi} + \frac{\delta\phi}{1+\phi}.$$

This difference equation has a unique fixed point $\theta^* = \delta$. Because $\theta_0 > \delta$ by assumption, for all $t < \infty$ we have $\theta_t > \delta$. *Q.E.D.*

Proof of Proposition 7: The statement for q_L follows directly from Lemma 8, which itself does not require proof, and inverting the $q_L^{**}(p)$ mapping. The other conditions follow analogously. For further details, refer to the similar computations in the body text of Acemoglu and Robinson (2000). *Q.E.D.*